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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 CAS Calculus, 2009

90833 Demonstrate an understanding of calculus concepts when solving differentiation and integration problems

Credits: Seven
2.00 pm Thursday 26 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Demonstrate an understanding of calculus concepts when solving problems. ☐	Demonstrate a deeper understanding of calculus concepts when solving problems. ☐	Demonstrate a comprehensive understanding of calculus concepts when solving problems. ☐
Overall Level of Performance ☐		

You are advised to spend 65 minutes answering the questions in this booklet.

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QUESTION ONE

(a) A skateboard ramp can be modelled by the equation $y = \frac{1}{1+x}$ where $0 \leq x \leq 3$.

The skateboard ramp is supported by a tangential rail with a gradient of $\frac{-1}{4}$.

Find the equation of this support rail.

(b) Find the exact area between the curve $y = \frac{4}{5x-2}$, the x-axis and the lines $x = 2$ and $x = 6$.

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(c) Another skateboard ramp can be modelled by the equation

$y = 1 - \sin\left(\frac{\pi x}{2}\right)$ where $0 \leq x \leq 5$.

The maximum gradient the ramp can safely have is $\frac{3}{2}$.

Will this ramp be safe?

(d) Show that the gradient of the function $4xy^2 - 3y = 4x^2$ is $\frac{8x - 4y^2}{8xy - 3}$.

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(e) The skateboard ramp modelled by the equation $y = 1 - \sin\left(\frac{\pi x}{2}\right)$ where $0 \leq x \leq 5$ m, is 1.5 m wide.

It is made from solid concrete.

Find the volume of concrete in the ramp.

(f) Yet another skateboard ramp can be modelled by the function $y = \ln(4x^2 + 3)$, where $-3 \leq x \leq 3$.

Show that the skateboard ramp has two points of inflection, and explain the effect the points of inflection will have on the curvature of the ramp.

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QUESTION TWO

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(a) The photograph below shows the Gateway Arch in St Louis.
It is 192 m across at the base and 192 m in height at the centre point.

http://upload.wikimedia.org/wikipedia/commons/archive/b/b9/20080325005332!St_Louis_Gateway_Arch.jpg

The height of the arch is measured every 32 m starting from one side of the arch.

The heights are given in the table below:

***x* = distance (m)**	0	32	64	96	128	160	192
***y* = height (m)**	0	85	155	192	155	85	0

Use Simpson's Rule, with 6 sub-intervals, to calculate an approximation for the area under the arch.

(b) The shaded region below is bounded by the curve $y = 20 - x^2$, the x-axis, the y-axis and the line $x = 4$.

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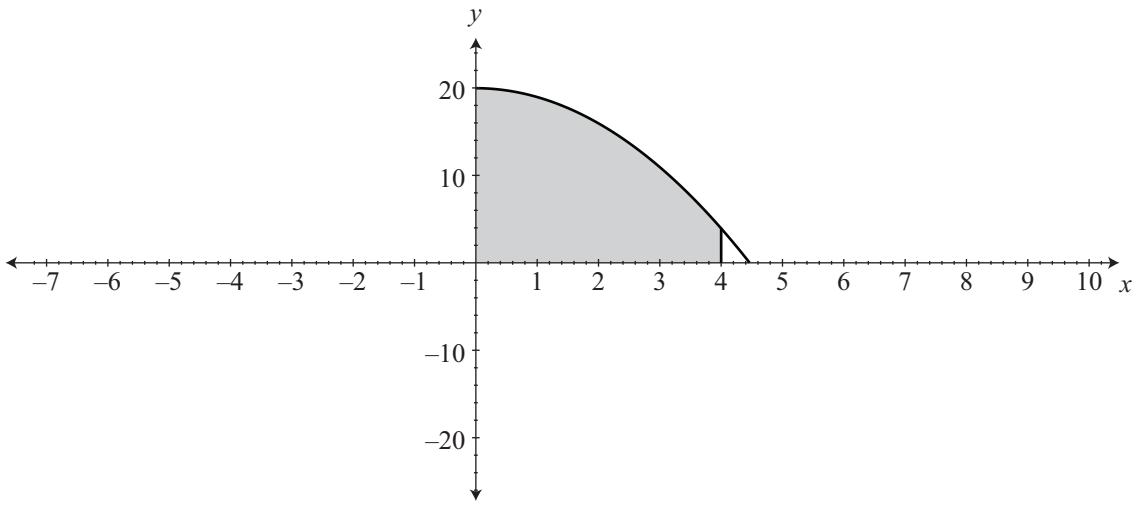

Calculate the volume of the solid generated if this region is rotated around the x-axis.

(c) Find the x coordinates of all the stationary points on the curve $y = x^2e^x$.

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(d) If $\frac{dy}{dx} = \frac{5x - 3}{e^{2y}}$

and $y = 0$ when $x = 1$, find the value of y when $x = 8$.

(e) An isosceles triangle is shown below.

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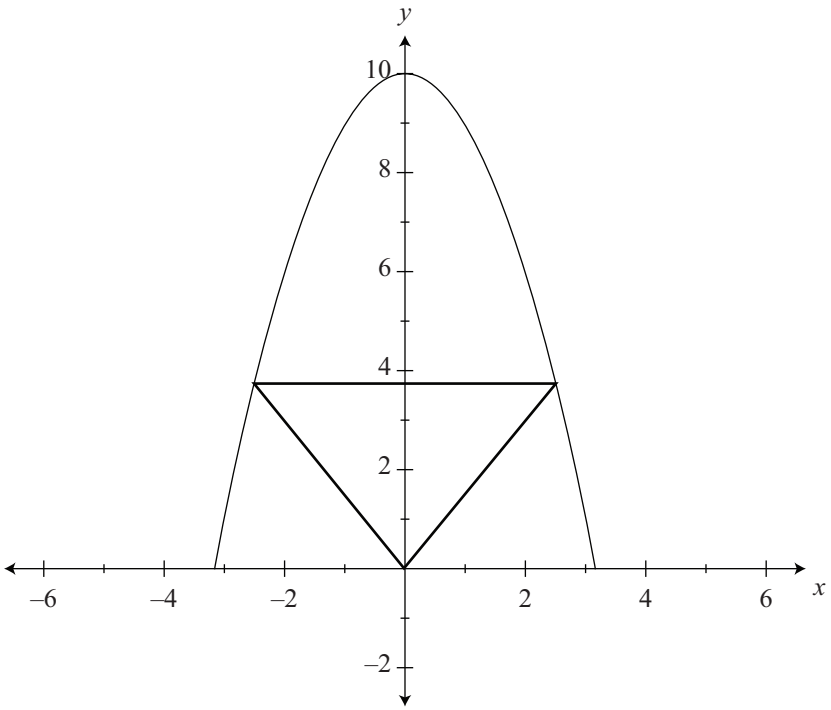

One vertex of the triangle is at (0,0).
The other two vertices are on the curve $y = 10 - x^2$, both vertices having the same y coordinate.

Find the maximum possible area, A, of such a triangle.

You may assume that $\frac{d^2A}{dx^2} < 0$.

(f)

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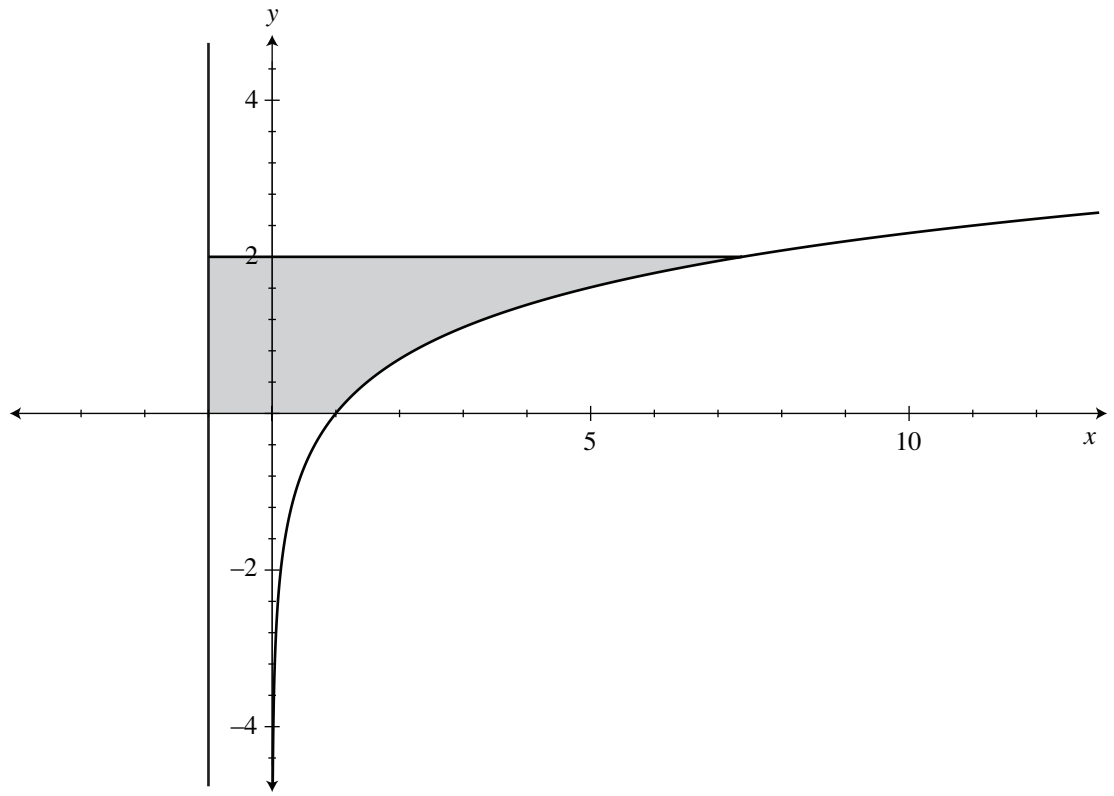

The shaded region above is bounded by the curve $y = \ln x$, the line $x = -1$, the x-axis and the line $y = 2$.

Calculate the volume of the solid generated if this region is rotated around the line $x = -1$.

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Extra paper for continuation of answers if required.
Clearly number the question.

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Question number

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